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THE SOLUTION OF NONLINEAR FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS BY USING SHIFTED CHEBYSHEV POLYNOMIALS METHOD AND ADOMIAN DECOMPOSITION METHOD

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ABSTRACT

In this paper we compare Adomian decomposition method (ADM) and Shifted Chebyshev polynomials method in order to obtain an approximate solution of nonlinear fractional integro-differential equations of Volterra and Fredholm integro-differential equations .We present some examples to find out accuracy of each method.

KEYWORDS: Fractional Integro-Di_Erential Equations, Caputo Derivative, Adomian Decomposition Method, Shifted Chebyshev Polynomials Method

1. INTRODUCTION

In recent years, there has been continuously renewed interest in integro-differential equations. Many mathematical models of physical phenomena produce integro-differential equations as fluid dynamics, biological models, and chemical kinetics, ([1],[3],[5]). Nevertheless, the development of the theory of derivatives and integral is due to Euler, Liouville and Abel (1823). However during the last ten years fractional calculus starts to attract much more attention of physicists and mathematicians,([7],[11],[14]). Some scientists and researchers interested in searching for method to approximate numerical for getting solution of the integro-differential equations of order fractional such as, Adomian decomposition method has been widely used by many researchers to solve the problems in applied sciences (Adomian 1944; Adomian 1989; Kaya and El-sayed 2003). Decomposition method provides an analytical approximation to linear and nonlinear problems.

In this method the solution is considered as the sum of an infinite series, rapidly converging to an accurate solution Shifted Chebyshev polynomials is applied for solving fractional integro-differential of equations the following form:

$$D^{\alpha}y(x) = g(x) + \lambda \int_0^x k(x,t)\, F(y(t))\, dt, \tag{1.1}$$

And

$$I^{\alpha}y(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} y(t)\, dt, \quad \alpha > 0, \quad x > 0\,. \tag{2.1}$$

for x; t 2 [0; 1], λ is a numerical, where the function g(x), k(x; t) are known and y(x) is the unknown function, D^{α} is Caputo fractional derivative and α is a parameter describing the order of the fractional derivative and F(y(x)) is a nonlinear continuous function.

2. BASIC DEFINITIONS

In this section we present some basic definitions and properties of the fractional calculus theory, which are used in this paper.

2.1 Definition

The Rieman-Liouvill fractional of α ≥0 is defined as,[8].

$$I^{\alpha}y(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}y(t)\,dt, \quad \alpha > 0, \quad x > 0. \tag{2.1}$$

It has the following properties:

$$I^0 y(x) = y(x), \tag{2.2}$$

Where I0 = I (Identity operator)

$$I^{\alpha}I^{\beta} = I^{\alpha+\beta}, \quad for\ all\ \ \alpha,\beta \geq 0. \tag{2.3}$$

2.2 Definition

The Caputo Definition of fractional derivative operator is given by:

$$D^{\alpha}y(x) = \begin{cases} J^{m-\alpha}y^{m}(x), & m-1 < \alpha \leq m\ ,\ m \in \mathbf{N}\ , \\ \frac{D^m y(x)}{Dx^m}\ , & \alpha = m. \end{cases} \tag{2.4}$$

It has the following properties:

$$D^{\alpha}I^{\alpha}y(x) = y(x), \tag{2.5}$$

$$I^{\alpha}D^{\alpha}y(x) = y(x) - \sum_{k=0}^{m-1} y^{(k)}(0^{+})\frac{x^k}{k!}\ ,\ \ x > 0. \tag{2.6}$$

$$D^{\alpha}x^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1-\alpha)}x^{\gamma-\alpha}, \tag{2.7}$$

For x > 0; α ≥ 0; γ> -1

Caputo fractional differentiation is a linear operation, similar to inter order differentiation.

$D^{\alpha}[\lambda y(x) + \mu g(x)] = \lambda D^{\alpha}y(x) + \mu D^{\alpha}g(x)$, where λ and μ are constants,([10],[15]).

3. NUMERICAL SCHEME

In this section Adomian decomposition method and Shifted Chebyshev polynomials are applied for solving nonlinear fractional integro-differential equations.

Adomain Decomposition Method

Consider equations (1.1) and (1.2) where D^{α} is the operator defined by (2.4) and (2.1) operating with I^{α} on both sides of equations (1.1) and (1.2) with obtain:

$$D^{\alpha}[\lambda y(x)+\mu g(x)]=\lambda D^{\alpha}y(x)+\mu D^{\alpha}g(x), \tag{3.1}$$

And

$$y(x)=\sum_{k=0}^{m-1} y^{(k)}(0^{+})\frac{x^{k}}{k!}+I^{\alpha}(g(x)+\lambda\int_{0}^{x}k(x,t)F(y(t))\,dt, \tag{3.2}$$

Adomain method defines the solution y(x) by the series, ([2],[12],[13])

$$y=\sum_{n=0}^{\infty} y_{n}, \tag{3.3}$$

And the nonlinear function F is decomposed as:

$$F=\sum_{n=0}^{\infty} A_{n}\,, \tag{3.4}$$

Where, A_n are the adomian polynomials given by:

$$A_{n}=\frac{1}{n!}[\frac{d^{n}}{d\lambda^{n}}F(\sum_{i=0}^{\infty}\lambda^{i}y_{i})]_{\lambda=0}, \qquad n=0,1,2,\ldots \quad . \tag{3.5}$$

The components y_0, y_1, y_2….. are determined recursively by:

$$y_{0}(x)=\sum_{k=0}^{m-1} y^{(k)}(0^{+})\frac{x^{k}}{k}+I^{\alpha}(y(x)), \tag{3.6}$$

$$y_{k+1}(x)=I^{\alpha}(g(x))+I^{\alpha}(\int_{0}^{x}k(x,t)A_{k}(t)\,dt), \tag{3.7}$$

$$y_{k+1}(x) = I^{\alpha}(g(x)) + I^{\alpha}(\int_0^1 k(x,t)A_k(t)\,dt). \tag{3.8}$$

Having defined the components y_o, y_1, y_2 … the solution y in series form defined by (3.3) follows immediately.

Shifted Chebyshev polynomials method

Shifted chebyshev polynomials is applied to study the numerical solution of nonlinear fractional integro-differenetial equations.

This method is based on approximating the unknown function y(x) as:

$$y_n(x) = \sum_{i=0}^{n} a_i T_i^*(x), \qquad 0 \le x \le 1, \tag{3.9}$$

a_i, i=1,2…. are constants.

Where $T_i^*(x)$ is the shifted chebyshev polynomials of first kind which is defined in terms of the Chebyshev polynomials $T_n^*(x)$ by the following relation, ([6],[9]).

$$T_n^*(x) = T_n^*(2x-1)\ , \tag{3.10}$$

And the following recurrence formulae:

$$T_n^*(x) = 2(2x-1)T_{n-1}^*(x) - T_{n-2}^*(x)\ ,\ n = 2,3...\ , \tag{3.11}$$

With the initial conditions

$$T_0^*(x) = 1 \quad , \quad T_1^*(x) = 2x - 1\ , \tag{3.12}$$

And the collocations points

$$x_j = \frac{1}{2}[1 + cos(\frac{j\pi}{n})]\ ,\ j = 0,\ 1,\ ...\, n\,. \tag{3.13}$$

Substituting from (3.9) into (1.1) and (1.2) we obtain

$$D^{\alpha}(\sum_{i=0}^{n} a_i T_i^*(x)) = g(x) + \lambda \int_0^x k(x,t)\,(\sum_{i=0}^{n} a_i T_i^*(t))\,dt\ , \tag{3.14}$$

and

$$D^{\alpha}(\sum_{i=0}^{n} a_i T_i^*(x)) = g(x) + \lambda \int_0^1 k(x,t)\,(\sum_{i=0}^{n} a_i T_i^*(t))\,dt\ . \tag{3.15}$$

Equation derived from (3.14) and (3.15) can be written as

$$\sum_{i=0}^{n} a_i D^{\alpha} T_i^*(x) = g(x) + \lambda \int_0^x k(x,t)\,(\sum_{i=0}^{n} a_i T_i^*(t))\,dt\ , \tag{3.16}$$

Similarly

$$\sum_{i=0}^{n} a_i D^{\alpha} T_i^*(x) = g(x) + \lambda \int_0^1 k(x,t)(\sum_{i=0}^{n} a_i T_i^*(t))\,dt\ . \tag{3.17}$$

Substituting from (3.13) into (3.16) and (3.17) we have:

$$\sum_{i=0}^{n} a_i D^{\alpha} T_i^*(x_j) = g(x_j) + \lambda \int_0^{x_j} k(x_j,t)(\sum_{i=0}^{n} a_i T_i^*(t))\,dt\ , \tag{3.18}$$

And

$$\sum_{i=0}^{n} a_i D^{\alpha} T_i^*(x_j) = g(x_j) + \lambda \int_0^1 k(x_j,t)(\sum_{i=0}^{n} a_i T_i^*(t))\,dt\ , \tag{3.19}$$

From (3.18) or (3.19) we obtain system of nonlinear algebraic equations, solving this system we obtain the values of the constant a_0, a_1, a_2…..substituting from these constants into (3.9), we obtain:

$$y(x) = a_0 T_0^*(x) + a_1 T_1^*(x) + a_2 T_2^*(x) + \dots\ \ . \tag{3.20}$$

4. NUMERICAL EXAMPLES

In this section we present some numerical example of nonlinear fractional integro-differential equations adomian decomposition method and shifted Chebyshev polynomials and compare the results.

Example 4.1

Consider the following nonlinear fractional integro-differential equation:

$$D^{0.75} y(x) = g(x) + \int_0^1 xt(y(x))^2\,dt, \tag{2.1}$$

And

$$g(x) = \frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{x}{8},$$

With the initial condition y(0)= 0 and the exact solution y(x)= x^3.

$$D^{3/4}y(x) = \frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{x}{8} + \int_0^1 xt(y(t))^2)\,dt,$$

$$y(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + I^{3/4}(\frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{x}{8}) + I^{3/4}(\int_0^1 xt(y(t))^2\,dt).$$

The Solution According to (ADM)

$$y_0(x) = y(0) + I^{3/4}(g(x)),$$

$$y_0(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + I^{3/4}(\frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{x}{8}), \tag{4.2}$$

$$y_0(x) = x^3 - 0.7771894662\,x^{7/4},$$

$$y_1(x) = I^{3/4}(\int_0^1 xt(A_0(t))\,dt, \tag{4.3}$$

$$y_1(x) = 0.06408417608\,x^{7/4}, \tag{4.4}$$

$$\ldots$$

$$y_{n+1}(x) = I^{3/4}(\int_0^1 (A_n(t))\,dt. \tag{4.5}$$

Then $y(x) \cong y_0(x) + y_1(x) + \ldots$,

The Solution According to Shifted Chebyshev Polynomials

$$y(x) \cong x^3 - 0.01363477054\,x^{7/4}, \tag{4.1}$$

When part of the truncated the summation in (3.9), taken n=3 and substituting in (4.1) we obtain

$$D^{3/4}y(x) = \frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{x}{8} + \int_0^1 xt(y(t))^2)\,dt, \tag{4.6}$$

By using the collocation point which is defined in (3.13) we obtain system of nonlinear algebraic equations:

$$D^{3/4}(\sum_{n=0}^{3} a_n T_n^*(x)) = \frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{x}{8} + \int_0^1 xt(\sum_{n=0}^{3} a_n T_n^*(t))^2\,dt, \tag{4.7}$$

Solving this system we obtain the values of the constant a_o= 5/16, a_1=3/16, a_3=1/32, substituting from those constant into (3.20).

$$\sum_{n=0}^{3} a_n\, D^{3/4}\, T_n^*(x_j) = \frac{64}{15}\frac{x_j^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{x_j}{8} + \int_0^1 x_j t(\sum_{n=0}^{3} a_n T_n^*(t))^2\,dt,$$

We obtain the approximate solution of equation (4.1) which is the same as the exact solution. Table 1 and figure 1 shows the comparison between shifted chebyshev polynomials method and (ADM) method.

$$y(x) = \frac{5}{16} + \frac{15}{32}(2x-1) + \frac{3}{16}(8x^2-8x+1) + \frac{1}{32}(32x^3-48x^2+18x-1) = x^3\,.$$

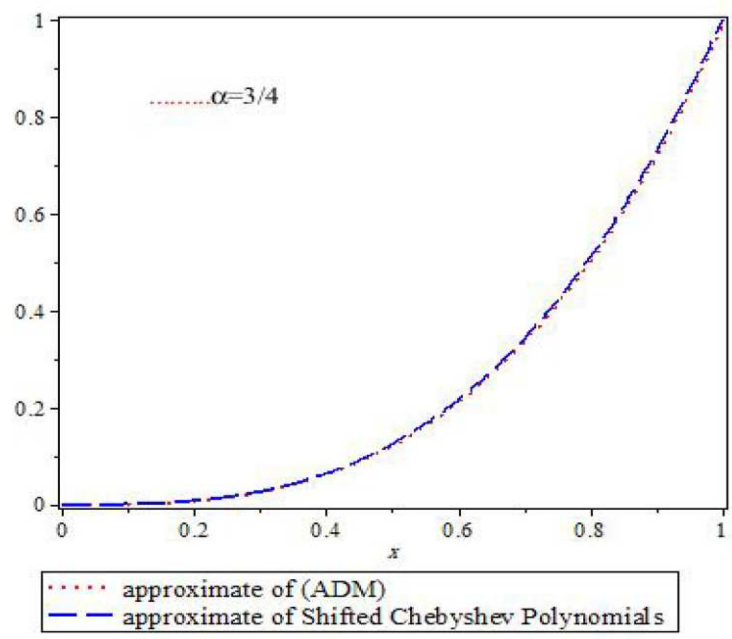

Figure 1: Numerical Results of Example 4.1.

Table 1: Indicate the Amount of Error in Example 4.1.

x	$exact = shifted\,chebyshev\,polynomials$	$Approximant\,by(ADM)$	$Error\,of(ADM)$
0.1	0.001	0.00075755356829	0.0002424643171
0.2	0.008	0.007184450500	0.000815549500
0.3	0.027	0.02534190264	0.00165809736
0.4	0.064	0.06125682940	0.00274317060
0.5	0.125	0.1209463585	0.0040536415
0.6	0.216	0.2104228475	0.0055771525
0.7	0.343	0.3356958547	0.0073041453
0.8	0.512	0.5027731107	0.0092268893
0.9	0.729	0.7176610661	0.0113389339
1	1	0.9863652295	0.0136347705

Example 4.2

Consider the following nonlinear fractional integro-differential equation, [4]:

$$D^{0.5}y(x) = \frac{8x^{1.5}}{3\sqrt{\pi}} - \frac{256x^{4.5}}{315\sqrt{\pi}} + I^{0.5}(y(t))^2 \ , \tag{4.8}$$

With the initial condition y(0)= 0 and the exact solution is y(x)= x^2.

The Solution According to (ADM)

$$D^{1/2}y(x) = \frac{8x^{3/2}}{3\Gamma(1/2)} - \frac{256x^{9/2}}{315\Gamma(1/2)} + I^{0.5}(y(t))^2 \ , \tag{4.9}$$

Using equation (2.1) in (4.9) we obtain:

$$D^{1/2}y(x) = \frac{8x^{3/2}}{3\Gamma(1/2)} - \frac{256x^{9/2}}{315\Gamma(1/2)} + \frac{1}{\Gamma(1/2)}\int_0^x (x-t)^{-1/2}(y(t))^2\,dt \, , \tag{4.10}$$

$$y(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + I^{1/2}(\frac{8x^{3/2}}{3\Gamma(1/2)} - \frac{256x^{9/2}}{315\Gamma(1/2)}) + I^{1/2}(\frac{1}{\Gamma(1/2)}\int_0^x (x-t)^{-1/2}(y(t))^2\,dt), \tag{4.11}$$

$$y_0(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + I^{1/2}(\frac{8x^{3/2}}{3\Gamma(1/2)} - \frac{256x^{9/2}}{315\Gamma(1/2)}) \, , \tag{4.12}$$

$$y_0(x) = -0.2x^5 + x^2,$$

$$y_1(x) = I^{1/2}(\frac{1}{\Gamma(1/2)}\int_0^x (x-t)^{-1/2}(A_0(t))\,dt) \, , \tag{4.13}$$

$$y_1(x) = -0.05x^8 + 0.2x^5 + 0.003636363635x^{11}.$$

$$y_2(x) = I^{1/2}\left(\frac{1}{\Gamma(1/2)}\int_0^x (x-t)^{-1/2}(A_1(t))\,dt\right), \tag{4.14}$$

$$y_2(x) = 0.05x^8 + 0.00008556149734x^{17} - 0.01636363636x^{11} + 0.001948051948x^{14},$$

$$y_{n+1}(x) = I^{1/2}\left(\frac{1}{\Gamma(1/2)}\int_0^x (x-t)^{-1/2}(A_n(t))\,dt\right). \tag{4.15}$$

Then

$$y(x) \cong y_0(x) + y_1(x) + y_2(x) + \dots ,$$

$$y(x) \cong -0.00008556149734x^{17} + 0.001948051948x^{14} - 0.01272727272x^{11} + x^2,$$

Is the approximate solution

The Solution According to Shifted Chebyshev Polynomials

$$D^{1/2}y(x) = \frac{8x^{3/2}}{3\Gamma(1/2)} - \frac{256x^{9/2}}{315\Gamma(1/2)} + I^{0.5}(y(t))^2 , \tag{4.19}$$

Using equation (2.1) in the third reduction of right side in (4.9) we obtain:

$$D^{1/2}y(x) = \frac{8x^{3/2}}{3\Gamma(1/2)} - \frac{256x^{9/2}}{315\Gamma(1/2)} + \frac{1}{\Gamma(1/2)}\int_0^x (x-t)^{-1/2}(y(t))^2\,dt. \tag{4.10}$$

When part of the truncated the summation in (3.9), take n=2 and substituting in (4.11) we obtain

$$D^{1/2}\left(\sum_{n=0}^{2} a_n T_n^*(x)\right)\frac{8x^{3/2}}{3\Gamma(1/2)} - \frac{256x^{9/2}}{315\Gamma(1/2)} + \frac{1}{\Gamma(1/2)}\int_0^x (x-t)^{-1/2}\left(\sum_{n=0}^{2} a_n T_n^*(t)\right)^2 dt. \tag{4.16}$$

By using the collocation point which is defined in (3.13) we obtain system of nonlinear algebraic equations

$$\sum_{n=0}^{2} a_n\, D^{1/2}\, T_n^*(x_j) = \frac{8x_j^{3/2}}{3\Gamma(1/2)} - \frac{256x_j^{9/2}}{315\Gamma(1/2)} + \frac{1}{\Gamma(1/2)}\int_0^{x_j} (x_j-t)^{-1/2}\left(\sum_{n=0}^{2} a_n T_n^*(t)\right)^2 dt. \tag{4.17}$$

Solving this system we obtain the values of the constant a_0=8/3, a_1=1/2, a_2=1/8, substituting from those constant into (3.20).

$$y(x) = \frac{8}{3} + \frac{1}{2}(2x-1) + \frac{1}{8}(8x^2 - 8x + 1) = x^2 \,,$$

We obtain the approximate solution of equation (4.9) which is the same as the exact solution. Table 2 and figure 2 shows the comparison between shifted chebyshev polynomials method and (ADM) method.

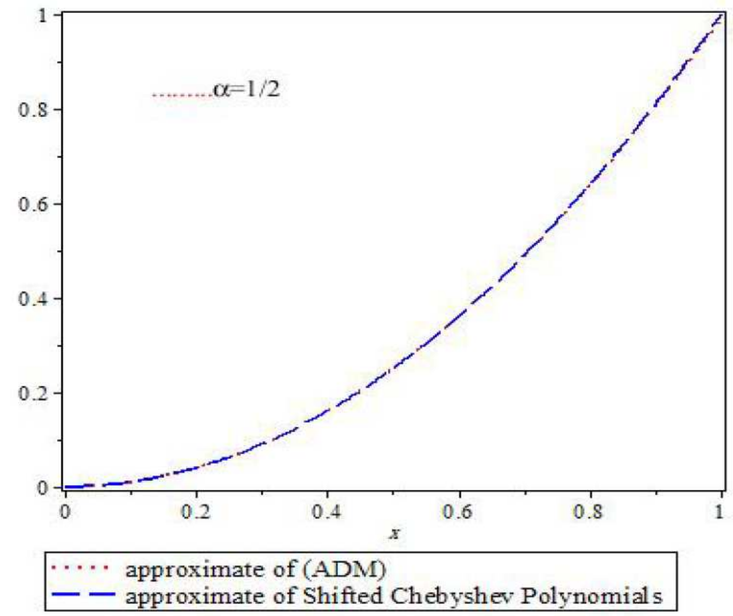

Figure 2: Numerical Results of Example 4.2.

Table 2: Indicate the Amount of Error in Example 4.2

x	$exact = shifted\ chebyshev\ polynomials$	$Approximant\ by(ADM)$	$Error\ of(ADM)$
0.1	0.01	0.01	0
0.2	0.04	0.3999999974	2.6×10^{-10}
0.3	0.09	0.08999997754	2.246×10^{-8}
0.4	0.16	0.1599994714	5.286×10^{-7}
0.5	0.25	0.2499939037	0.0000060963
0.6	0.36	0.3599553379	0.0000446621
0.7	0.49	0.4897613533	0.0002386467
0.8	0.64	0.6389904851	0.0010095149
0.9	0.81	0.8064374296	0.0035625704
1	1	0.9891352177	0.0108647823

Example 4.3

Consider the following nonlinear fractional integro-differential equation:

$$D^{0.25}y(x) = \frac{4}{3}\frac{x^{0.75}}{\Gamma(0.75)} - \frac{x}{4} + \int_0^1 xt\,(y(t))^2\,dt \,, \tag{4.18}$$

With the initial condition y(0)= 0 and the exact solution y(x)= x.

The solution according to (ADM):

$$D^{1/4}y(x) = \frac{4}{3}\frac{x^{3/4}}{\Gamma(3/4)} - \frac{x}{4} + \int_0^1 xt\,(y(t))^2\,dt\,,$$

$$y(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + I^{1/4}(\frac{4}{3}\frac{x^{3/4}}{\Gamma(3/4)} - \frac{x}{4}) + I^{1/4}(\int_0^1 xt\,(y(t))^2\,dt)\,,$$

$$y_0(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + I^{1/4}(\frac{4}{3}\frac{x^{3/4}}{\Gamma(3/4)} - \frac{x}{4})\,,$$

$$y_0(x) = -0.2206525301x^{5/4} + 0.9999999999x\ .$$

$$y_1(x) = I^{1/4}(\int_0^1 xt(A_0(t))\,dt)\,,$$

$$y_1(x) = 0.1385547567x^{5/4}.$$

$$y_2(x) = I^{1/4}(\int_0^1 xt(A_1(t))\,dt)\,,$$

$$y_2(x) = 0.04555546200x^{5/4}.$$

$$y_3(x) = I^{1/4}(\int_0^1 xt(A_2(t))\,dt)\,,$$

$$y_3(x) = 0.01874349194x^{5/4}.$$

$$\cdots$$

$$y_{n+1}(x) = I^{1/4}(\int_0^1 xt(A_n(t))\,dt)\,,$$

$$Then\ \ y(x) \cong y_0(x) + y_1(x) + y_2(x) + y_3(x) + \dots\ .$$

$y(x) \cong 0.9999999999x - 0.01779881946x^{5/4}$, is the approximate solution.

The Solution According to Shifted Chebyshev Polynomials

$$D^{1/4}y(x) = \frac{4}{3}\frac{x^{3/4}}{\Gamma(3/4)} - \frac{x}{4} + \int_0^1 xt\,(y(t))^2\,dt\,, \tag{4.19}$$

When part of the truncated the summation in (3.9), taken n=2 and substituting in (4.19) we obtain

$$D^{1/4}(\sum_{n=0}^{2} a_n T_n^*(x)) = \frac{4}{3}\frac{x^{3/4}}{\Gamma(3/4)} - \frac{x}{4} + \int_0^1 xt\,(\sum_{n=0}^{2} a_n \tau_n^*(t))^2\,dt\,. \tag{4.26}$$

By using the collocation point which is defined in (3.13) we obtain system of nonlinear algebraic equations,

$$\sum_{n=0}^{2} a_n\, D^{1/4}\, T_n^*(x_j) = \frac{4}{3}\frac{x_j^{3/4}}{\Gamma(3/4)} - \frac{x_j}{4} + \int_0^1 x_j t\, (\sum_{n=0}^{2} a_n T_n^*(t))^2\, dt\,. \tag{4.27}$$

Solving this system we obtain the values of the constant a_0= 1/2, a_1= 1/2, a_2= 0, substituting from those constant into (3.20).

$$y(x) = \frac{1}{2} + \frac{1}{2}(2x-1) = x$$

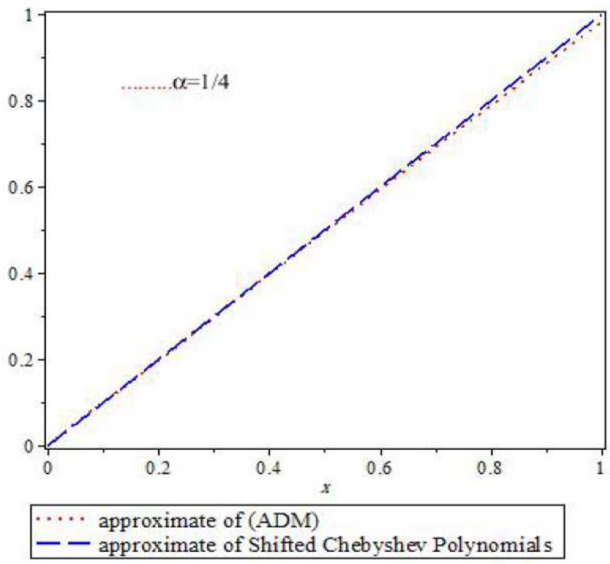

Figure 3: Numerical Results of Example 4.3.

Table 3 and figure 3 shows the comparison between shifted chebyshec polynomials method and (ADM) method.

Table 3: Indicate the Amount of Error in Example 4:3

x	$exact = shifted\,chebyshev\,polynomials$	$Approximant\,by(ADM)$	$Error\,of(ADM)$
0.1	0.1	0.9899909882	0.00100090118
0.2	0.2	0.1976194424	0.0023805576
0.3	0.3	0.2960482199	0.0039517801
0.4	0.4	0.3943380480	0.0056619520
0.5	0.5	0.4925165183	0.0074834817
0.6	0.6	0.5906010300	0.0093989700
0.7	0.7	0.6886037032	0.113962968
0.8	0.8	0.7865335326	0.0134664674
0.9	0.9	0.8843974948	0.0156025052
1	1	0.9822011804	0.0177988196

CONCLUSIONS

From solving examples we find that Shifted Chebyshev polynomials method is better than Adomian decomposition method, the results obtain by using Maple 16.

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